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When $n = 1$, $U_1 = 1$, and when $n = 2$, $U_2 = 4$; then (5) gives for determining C_1 and C_2,

$$C_1(2 + \sqrt{3}) + C_2(2 - \sqrt{3}) = 1,$$
$$C_1(2 + \sqrt{3})^2 + C_2(2 - \sqrt{3})^2 = 4,$$

giving $C_1 = \frac{1}{6}\sqrt{3}$ and $C_2 = -\frac{1}{6}\sqrt{3}$.

(1) then becomes $$U_n = \tfrac{1}{6}\sqrt{3}\{(2 + \sqrt{3})^n - (2 - \sqrt{3})^n\},$$

the general term. The usual theory for the sum of n terms gives

$$S_n = C + C_1 m_1 \frac{m_1^n}{m_1 - 1} + C_2 m_2 \frac{m_2^n}{m_2 - 1}.$$

Substituting the values of m_1, m_2, C_1, C_2, we have

(2) $$S_n = C + \tfrac{1}{6}\sqrt{3}\left\{ \frac{(2 + \sqrt{3})^{n+1}}{\sqrt{3} + 1} + \frac{(2 - \sqrt{3})^{n+1}}{\sqrt{3} - 1} \right\}.$$

When $n = 1$, $S_n = 1$, and (2) gives $C = -\frac{1}{2}$, and this in (2) gives the required sum.

Also solved by Amelia Benson, G. W. Hartwell, E. B. Escott, A. M. Harding, N. P. Pandya, and O. S. Adams.

GEOMETRY.

489. Proposed by NATHAN ALTSHILLER, The University of Oklahoma.

The parallels to the asymptotes a, b of a given hyperbola, drawn from a variable point of the curve, meet a and b in P, Q respectively. The line PQ envelops an hyperbola whose asymptotes are a and b.

I. Solution by E. J. Oglesby, Williamsburg, Virginia.

Take the asymptotes a, b as the axes of coördinates. Then the equation of the hyperbola may be taken as $xy = c^2$ and the coördinates of the variable point on the hyperbola as $(ct, c/t)$ in terms of the parameter t. P is the point $(ct, 0)$, and Q is $(0, c/t)$.

The equation of PQ may be written

(1) $$t^2 y - ct + x = 0.$$

We find the envelope of (1) by applying the condition that this equation shall have equal roots in the parameter t.

Hence, we have

$$(-c)^2 - 4yx = 0, \quad \text{or} \quad xy = c^2/4,$$

which is an hyperbola having a and b as asymptotes.

II. Solution by the Proposer.

The tangents a, b to a given conic at the points A, B, are met by the lines BM, AM joining A and B to a variable point M of the curve, in the points P, Q respectively. The line PQ envelops a conic having a double contact with the given curve at the points A, B.

Indeed, the lines AM, BM describe two projective pencils, hence their sections by the lines a, b are two projective ranges.

$$(P\cdots) \,\overline{\wedge}\, B(M\cdots) \,\wedge\, A(M\cdots) \,\overline{\wedge}\, (Q\cdots).$$

Consequently the line PQ envelops a conic tangent to a and b. To the point (ab) considered as an element of a and b in turn correspond, in the ranges $(Q\cdots)$ and $(P\cdots)$, the points B and A, these points are therefore the points of contact of a and b with the envelope, which proves the proposition.

If for A, B are taken some remarkable points of the conic, special cases of this general proposition are obtained. For example, if A be the point at infinity of a parabola, and B its vertex, the

proposition takes the following form: *The diameter passing through a variable point of a parabola, meets the tangent at the vertex in the point P. The parallel through P to the line joining M to the vertex of the parabola, envelops another parabola having the same vertex and the same axis as the given curve.*

The proposed problem is another special case of this general proposition, namely when both A and B are at infinity.

The duals of the three propositions are, in order:

The points of intersection of two fixed tangents to a given conic, with a variable tangent to the same curve, are projected from the points where the fixed tangents touch the conic. The point of intersection of the two projecting lines describes a conic having a double contact with the given curve.

From a variable point of the tangent at the vertex of a given parabola, are drawn the diameter and the tangent to the curve. The point of intersection of the diameter with the parallel to the tangent through the vertex of the curve, describes a parabola having the same axis and the same vertex as the given curve.

The parallels to the asymptotes of a given hyperbola drawn through the points of intersection of the latter lines with a variable tangent to the curve, intersect in a point whose locus is an hyperbola having the same asymptotes as the given curve.

Also solved by O. S. ADAMS, CLARA L. BACON, J. W. CLAWSON, A. M. HARDING, HORACE OLSON, PAUL CAPRON, G. W. HARTWELL, R. M. MATHEWS, and N. P. PANDYA.

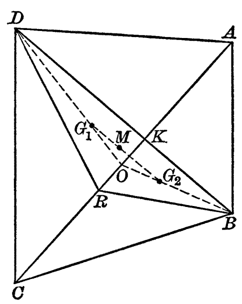
490. Proposed by ELMER E. MOOTS, University of Arizona.

In any quadrilateral $ABCD$, let AC and BD be the diagonals intersecting at K. On AC, lay off CR equal to AK. Join B and R. Connect the middle point G of BR with D. On GD lay off GM equal to $\frac{1}{3}GD$. Show that M is the center of gravity of the quadrilateral.

SOLUTION BY A. M. HARDING, University of Arkansas.

It is evident that M is the center of gravity of the triangle BDR. Hence it will be sufficient to prove that the triangle BDR and the quadrilateral $ABCD$ have the same center of gravity.

Let O be the mid-point of RK, then it will also be the mid-point of CA. Then G_1 is the center of gravity of the triangles RDK and CDA, and G_2 is the center of gravity of the triangles RBK and CBA where $OG_1 = \frac{1}{3}OD$ and $OG_2 = \frac{1}{3}OB$.



Since

$$\frac{\triangle RDK}{\triangle RBK} = \frac{\triangle CDA}{\triangle CBA},$$

is follows that the center of gravity of the triangle BDR will also be the center of gravity of the quadrilateral $ABCD$.

Also solved by J. W. CLAWSON, O. S. ADAMS, and N. P. PANDYA.

491. Proposed by N. P. PANDYA, Sojitra, India.

In a triangle $mx = b$ and $nx = c$, determine a relation between m, n, x, A and s, and solve it for x.